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Fourth Semester B.E. Degree Examination, June/July 2011
Engineering Mathematics - IV

Time: 3 hrs.

Max. Marks:100

Note: Answer FIVE full questions, selecting atleast TWO questions each from Part - A and Part - B.

PART - A

- 1
 - a. Using Taylor's series method, find y at $x = 0.1$ and $x = 0.2$ considering upto 4th degree terms. Given that $\frac{dy}{dx} = x^2y - 1$ and $y(0) = 1$. (06 Marks)
 - b. Solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$, find y at $x=0.2$ using Runge - Kutta method of 4th order taking step - length $h = 0.2$ accurate upto 4th decimal place. (07 Marks)
 - c. Given that $\frac{dy}{dx} = x^2(1+y)$ and $y(1) = 1$; $y(1.1) = 1.233$; $y(1.2) = 1.548$; $y(1.3) = 1.979$, find y at $x = 1.4$ using Adams - Bashforth predictor and corrector formula. (07 Marks)

- 2
 - a. Find Analytic function whose real part is $u = x^2 - y^2 + \frac{x}{x^2 + y^2}$. (06 Marks)
 - b. Under the transformation $W = e^Z$, prove that family of lines parallel to $y -$ axis in $Z -$ plane transforms into family of concentric circles in $W -$ plane. (07 Marks)
 - c. Find Bilinear transformation, that transforms $Z = -1, i, 1$ on to points $W = 1, i, -1$, in $W -$ plane respectively. Also find invariant points. (07 Marks)

- 3
 - a. Evaluate $\int_C \frac{e^{2Z}}{(Z+1)(Z+2)} dZ$, where 'C' is a circle $|Z| = 3$. (06 Marks)
 - b. Obtain the power series which represents the function $f(Z) = \frac{Z^2 - 1}{Z^2 + 5Z + 6}$ in the region $2 < |Z| < 3$. (07 Marks)
 - c. Using Cauchy's Residue theorem evaluate $\int_C \frac{2Z^2 + 1}{(Z+1)^2(Z-2)} dZ$, where 'C' is circle with $|Z| = 3$. (07 Marks)

- 4
 - a. Using Frobenius series solution method, solve $\frac{d^2y}{dx^2} + xy = 0$. (06 Marks)
 - b. Reduce the differential equation $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (k^2x^2 - n^2) y = 0$ into Bessel's form and write the complete solutions for n is not integral or zero. (07 Marks)
 - c. Express the polynomial $2x^3 - x^2 - 3x + 2$ in terms of Legendre's polynomial. (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identity, appeal to evaluator and/or equations written eg. 4, 5, 50, will be treated as malpractice.

PART – B

- 5 a. Fit the best possible curve of the form $y = a + bx$, using method of Least square for the data: (06 Marks)

X:	1	3	4	6	8	9	11	14
Y:	1	2	4	4	5	7	8	9

- b. The lines of regressions are $x + 2y = 5$ and $2x + 3y = 8$. Find i) means of the variables x and y ii) correlation coefficient between x and y . (07 Marks)
- c. Three typists A, B, C typed 50%, 30% and 20% of pages of a book. The percentage of defectively typed pages by them is 3, 4, 5 respectively. If a page is selected from the book at random, what is the probability that it is defectively typed and it is typed by 'A'? (07 Marks)
- 6 a. The random variable X has the following probability mass function

X:	0	1	2	3	4	5
P(X):	K	3K	5K	7K	9K	11K

- i) find K ii) find $P(X < 3)$ iii) find $P(3 < X \leq 5)$. (06 Marks)
- b. Alpha – particles are emitted by a radio active source at an average of 5 emissions in 20 minutes. What is the probability that there will be i) exactly two emissions ii) at least two emissions in 20 minutes? (07 Marks)
- c. A sample of 100 dry battery cells tested to find the length of life produced by a company and following results are recorded : mean life = 12 hours, standard deviation = 3 hours. Assuming data to be normally distributed, find the expected life of a dry cell :
i) have more than 15 hours ii) between 10 and 14 hours. (07 Marks)
- 7 a. Explain the following terms : i) Null hypothesis ii) Standard error iii) Test of significance. (06 Marks)
- b. Find the range of number of heads out of 64 tosses of a coin which will ensure fairness of coin at 5% level of significance using binomial distribution. (07 Marks)
- c. A survey conducted on 64 families with 3 children each and recorded as follows :

No. of Male children :	0	1	2	3
No. of families :	6	19	29	10

Apply Chi – Square test to test whether male and female children are equiprobable at 5% level of significance. (07 Marks)

- 8 a. The Joint probability distribution of two Random variable X and Y are given as :

Y \ X	1	3	9
2	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$
4	$\frac{1}{4}$	$\frac{1}{4}$	0
6	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$

- i) find Marginal distribution of X and Y ii) find $COV(X, Y)$. (06 Marks)
- b. Find the unique fixed probability vector of the regular stochastic matrix.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

(07 Marks)

- c. A player's luck follows a pattern. If he wins a game the probability of winning next game is 0.6. However if he loses the game the probability of losing the next game is 0.7. There is an even chance of winning the first game. If so i) what is the probability of winning 2nd game ii) What is the probability of winning 3rd game? (07 Marks)
